Sequences Masterclass: Exercises

1. For the following sequences, find the *Recursive Relationship* and the *General Formula*.

   (a) $6,\ 10,\ 14,\ 18,\ 22,\ 26,\ \ldots$

   (b) $81,\ 27,\ 9,\ 3,\ 1,\ \frac{1}{3},\ \ldots$

   (c*) $5,\ 6,\ 4,\ 7,\ 3,\ 8,\ \ldots$

   (d*) $5,\ 6,\ 4,\ \frac{7}{3},\ \frac{19}{12},\ \frac{79}{60},\ \ldots$

2. Follow these steps:

   i. Pick a positive integer N.

   ii. Start a sequence with two integer values a and b where $a \leq b$.

   $$a, \quad b$$

   iii. Let the next term of the sequence be N times the second then added to the first

   $$a, \quad b, \quad a + bN$$

   iv. Repeat the process with the next terms until you obtain 2 numbers over $N \times 1000$.

   v. Divide the last term by the term before and denote that term as δ.

   > i. $N = 7$.
   >
   > ii. Pick $a = 4$ and $b = 5$, then the sequence starts as
   >
   > $$4, \quad 5$$
   >
   > iii. The next term in the sequence is $4 + 7 \times 5 = 39$, so the sequence is now
   >
   > $$4, \quad 5, \quad 39$$
   >
   > iv. The sequence is
   >
   > $$4, \quad 5, \quad 39, \quad 278, \quad 1985, \quad 14173, \quad 101196$$
   >
   > v.
   >
   > $$\frac{101196}{14173} = 7.14\ldots$$

   Show that the value of δ you obtained satisfies

   (I) $\delta = N + \frac{1}{\delta}$ (II) $\delta = \sqrt{1 + N\sqrt{1 + N\sqrt{1 + N\sqrt{1 + \ldots}}}}$ (III) $\delta = N + \cfrac{1}{N + \cfrac{1}{N + \cfrac{1}{N + \cfrac{1}{N + \ldots}}}}$

3. * For some value N, show that if some number δ satisfies

$$\delta = \sqrt{1 + N\sqrt{1 + N\sqrt{1 + N\sqrt{1 + \dots}}}} \quad \text{then} \quad \delta^2 - N\delta - 1 = 0.$$

4. * For some value N, show that if some number δ satisfies

$$\delta = N + \frac{1}{N + \frac{1}{N + \frac{1}{N + \frac{1}{N + \dots}}}} \quad \text{then} \quad \delta^2 - N\delta - 1 = 0.$$

Answers

1. For $n = 1, 2, 3, \dots$:

(a) Recursive: $a_{n+1} = a_n + 4, \quad a_1 = 6$ General: $a_n = 6 + 4(n-1)$

(b) Recursive: $a_{n+1} = \frac{a_n}{3}, \quad a_1 = 81$ General: $a_n = \frac{81}{3^{n-1}}$

(c*) Recursive: $a_{n+1} = a_n - (-1)^n n, \quad a_1 = 5$ General: $a_n = 5 + \frac{2n(-1)^n + 1 - (-1)^n}{4}$

(d*) Recursive: $a_{n+1} = \frac{a_n + n}{n}, \quad a_1 = 5$ General: $a_n = \frac{1}{(n-1)!}\left(5 + \sum_{i=1}^{n-1} i!\right)$